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Model for Evaluating the Cost Consequences of Deferring New System Acquisition Through Upgrades

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PREFACE

The work documented in this paper was performed by IDA as an internal Central Research Project. The analyses presented herein grew out of and clarified ideas introduced in a study entitled *C-130 Remanufacturing Study*,[1] conducted for the Office of the Secretary of Defense (Acquisition and Technology).

We address the general problem of deciding whether to upgrade an existing system that is aging or to replace it with a new one. The methodology developed in this paper is equally applicable to any system, military or otherwise, and not just to C-130 modernization. Upgrading does not always avoid replacement entirely, but does defer it to the future where discounting reduces the ultimate acquisition cost. On the other hand, upgrading comes with a cost of its own and generally entails higher operating costs than a new replacement system.

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[1] *C-130 Remanufacturing Study*, IDA Paper P-3404, June 1998, UNCLASSIFIED

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MODEL FOR EVALUATING THE COST CONSEQUENCES OF DEFERRING NEW SYSTEM ACQUISITION THROUGH UPGRADES

This paper addresses a critical question confronting the Department of Defense (DoD) in a time of reduced budgets. The DoD needs to continually modernize the military forces under its authority but must do so with steadily reduced funding support. The question for any system under consideration is between upgrading and replacement. Some systems cannot be modernized without complete replacement. Stealth shaping of aircraft fuselages to produce less vulnerable aircraft may be one such example. Many others can be modernized through the installation of upgrades to existing systems. Airlift aircraft may serve as an example of the latter. In general, upgrading is less costly in the near term, but is only reasonable if the system that is being upgraded can perform well enough and last long enough. And even with upgrades, if the life cycle period under consideration is long enough, eventually the upgraded system will need replacement, so acquisition is deferred, not avoided.

In this paper we consider the problem of determining whether to upgrade an existing aging system or to replace it with a new one. This issue arose when we were assisting the Office of the Secretary of Defense (OSD) to determine how to best modernize C-130 cargo/transport aircraft. There are proponents for both replacing older aircraft and upgrading them. Some existing aircraft are more than 30 years old and require significant upgrades to meet reliability, performance, and safety requirements. New C-130 aircraft will presumably cost less to operate because of higher reliability and lower maintenance costs, but come with a high acquisition price.

We first develop the general formalism appropriate to answering the questions: (1) buy now or (2) upgrade first and, if necessary, buy later? The formalism is sufficiently general that it applies equally to trucks, ships, and aircraft. The approach begins with simple assumptions—constant costs, a homogeneous population, and equal performance levels. The latter sections extend the formalism to include such issues as time-dependent costs, heterogeneous populations, performance differences, and risk.

The two alternatives considered are as follows:

1. **Upgrade-First**: With this alternative, an existing system is upgraded now to extend its service life; at a specific time in the future, a time treated initially as a variable, it may be replaced by a new system. The major benefit is to delay the large expenditure for a replacement system.
2. **Replace-Now**: With this alternative, the existing system is replaced by a new system now. The major benefits are avoiding the upgrade cost and realizing cumulative economies in operating costs.

Note that continued operation with the existing system is not considered as a *third* option. Continued operation of the existing system may not be viable because of aging, performance, safety, or commonality issues. However, if these impediments do not exist, the model includes continued operation of the current system as a special case of the Upgrade-First option, for which there is no cost for upgrading.

At this point, we shall assume that if there is a population of systems, it has sufficient homogeneity with respect to the relevant characteristics to allow us to consider only a single system in the analysis. Heterogeneous population issues are treated later.

A. DEFINITIONS AND NOTATION

The following terms and definitions are used in this paper:

Life cycle period—The number of years of operation used in the analysis.

Life cycle cost (LCC—The total cost associated with acquiring, operating, and disposing of systems over the life cycle period.

Service life (SL)—The expected duration of usable service to be provided by a system; typically, units are in such terms as operating hours, miles, and cycles. For this paper, we use the more generic measure, age, measured in years. Given an average usage per year (e.g., miles driven per year), one can easily convert age to a more direct measure of system service life.

Existing system—A system currently in service.

Upgraded system—An existing system that has been upgraded to extend its service life and which meets current requirements.

Replacement system—A new system that replaces an existing or upgraded system.

Residual value—The worth of a system at the time it is removed from service or at the end of the life cycle period.

We define the following variables. In all cases, the costs are expressed in constant year dollars:

A_u = cost to upgrade an existing system

A_r = cost to acquire a new system to replace an existing system

C_u = annual cost to operate an upgraded system

C_r = annual cost to operate a replacement system

$R_e(y)$ = residual value of an existing system y years old

$R_u(y)$ = residual value of an existing upgraded system y years after the upgrade

$R_r(y)$ = residual value of a new system y years after purchase

n = number of years an upgraded system is operated before it is replaced with a new system

d = discount factor [$=1/(1+i)$, where $i > 0$ is the interest or discount rate]

L = life cycle period, measured in number of years.

$LCC_u(L)$ = life cycle cost over L years when an existing system is upgraded now and operated for L years

$LCC_u(L, n)$ = life cycle cost over L years when an existing system is upgraded now and operated for n years before it is replaced by a new system

$LCC_r(L)$ = life cycle cost for L years when an existing system is replaced now.

We note below some assumed relationships between the variables:

$A_r > A_u$, the constant-year cost to acquire a new system is greater than the cost to upgrade the existing system

$C_u > C_r$, the annual operating cost of an upgraded system is greater than that of a replacement system

$R_e(x) < R_u(x) < R_r(x)$, for the same time periods, the residual value of an existing system is less than that of an upgraded system, which, in turn, has a residual value less than a replacement system

$0<d<1$, since the discount rate is non-zero.

Finally, we note that we can represent the case for continued operation with the existing system by setting $A_u = 0$. If this is done, C_u would then represent the operational cost of the existing system, and there would be no distinction between the existing system residual value and that of the upgraded system, i.e., $R_e(y) = R_u(y)$.

B. EXAMPLE OF COST EXPENDITURE, DISCOUNTING, AND DECISION TIMING

Because we are dealing with cost streams where the key issue is delaying an expensive purchase through investing in an upgraded system now, we use a net present-value life cycle cost criterion. That is, all future costs (positive and negative) are discounted to the current year, which we define to be year 1. Discounting is the proper manner in which to treat future expenditures, since it is a measure of the value of deferring acquisitions to the future. It is through discounting that the benefits of deferred acquisition are measured quantitatively.[1]

Let us illustrate the general principles with a simple example. Table 1 illustrates the year-by-year cost, discounting, and residual value timing for a hypothetical case of upgrading the existing system in year 1 and then replacing it at the beginning of year 4. By definition, the value of n—the number of years an upgraded system operates before replacement—is 3. A 7-year life cycle period is assumed in this example. The activities and associated costs during each of the 7 years are shown in the table. For example, in the column labeled year 1, the system is upgraded at cost A_u for which the operating and support (O&S) costs are C_u. At the beginning of year 4 (equivalent to the end of year 3), a new system is bought at a cost of A_r and a new O&S cost C_r is invoked. Moreover, the old upgraded system is sold with an estimated residual value of $R_u(3)$, the value it has 3 years after upgrading.

We also assume that the upgrade and purchase actions take place at the beginning of the year and that operating costs are spread evenly throughout the year. We use a mid-year approximation for discounting the cost streams. Therefore, in the third year, the last one operating with the existing upgraded system, the operating costs are discounted over

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1 In accordance with OMB Circular A-94, *Guidelines and Discount Rates for Benefit-Cost Analysis of Federal Programs*, any analysis used to determine whether a Government program can be justified on economic principles must use net present value. For analyzing alternatives, the circular states, "A program is cost-effective if, on the basis of life cycle cost analysis of competing alternatives, it is determined to have the lowest costs expressed in present value terms for a given amount of benefits."

2.5 years. At the beginning of the fourth year when the new system is purchased, the acquisition costs are discounted for 3 years, but the operating costs, which occur over the year, are discounted for 3.5 years. When the replacement system is purchased, there is a credit for the residual value of the upgraded system, which has been operating for 3 years. At the end of the life cycle period, there is a residual value credit for the replacement system, which in this case has been operating for 4 years.

Table 1. Illustration of Timing and Discounting for Buying a New System After 3 Years

Year	1	2	$3 = n$	4	5	6	$7 = L$
Upgrade or Acquisition Cost	A_u			A_r			
Discount period, Acquisition	--			3 Yr.			
Operating Cost	C_u	C_u	C_u	C_r	C_r	C_r	C_r
Discount Period, Operation	0.5 Yr.	1.5 Yr.	2.5 Yr.	3.5 Yr.	4.5 Yr.	5.5 Yr.	6.5 Yr.
Residual Value				$R_u(3)$			$R_r(4)$
Discount Period, Residual Value				3 Yr.			7 yr.

C. THE LIFE CYCLE COST MODEL WITH NO RESIDUAL VALUE

We use a very simple form for life cycle cost—namely acquisition or upgrade cost plus yearly operating cost, discounted as appropriate. Issues such as development costs, training, reliability impacts, maintenance requirements, and parts and materials costs are assumed embodied in the acquisition/upgrade and operating costs. We also assume that acquisition and operating costs remain constant over time.[2] We defer consideration of residual values to a later section.

2 This assumption may not be valid. If the new systems are currently in production, then delaying their purchase may result in a cost increase, especially if the delay causes a production break. Similarly, operating costs may increase with time. This is more likely for the upgraded existing systems, some of which may have to undergo expensive structural repairs as they age. Incorporating time dependent cost functions (if known) is not difficult except that a closed form analytical solution may be obviated. We discuss this in more detail later.

1. No Future Replacement of Upgraded System

We first consider the simplest case, where it is assumed that the service life of the system after upgrade exceeds the sum of the system's current age and the life cycle period. That is, we expect that an upgraded system will not have to be replaced before L years after upgrade. If we denote the service life of the upgraded system by SL_u and the age at upgrade by G_u, both measured in years, we have this condition represented by the formula,

$$G_u + L < SL_u \,. \tag{1}$$

Through straightforward consideration we find that the life cycle cost for an upgraded system is

$$LCC_u(L) = A_u + C_u \sum_{k=1}^{L} d^{k-0.5} \,. \tag{2}$$

The first term represents the cost to upgrade. The second term is the discounted operating cost stream over the life cycle period, with C_u being spent each year.

The corresponding life cycle cost for the Replace-Now alternative is

$$LCC_r(L) = A_r + C_r \sum_{k=1}^{L} d^{k-0.5} \,. \tag{3}$$

If life cycle cost is the only basis for a decision between the two alternatives, one should choose the Upgrade-First option if the total life cycle cost for upgrading is less than that for replacement, i.e. if

$$LCC_u(L) < LCC_r(L) \tag{4}$$

or,

$$A_u + C_u \sum_{k=1}^{L} d^{k-0.5} < A_r + C_r \sum_{k=1}^{L} d^{k-0.5}. \tag{5}$$

Through use of the following identity for a geometric series,

$$\sum_{i=1}^{m} r^i = r\frac{1-r^m}{1-r}, \qquad r^2 < 1, \tag{6}$$

we find that the decision to upgrade first should be made if

$$A_r - A_u > (C_u - C_r)d^{0.5}\frac{1-d^L}{1-d}. \tag{7}$$

The left side of equation 7 is the difference in acquisition cost between the upgraded and replacement systems. The right side is the present value of the stream of operating cost differences.

2. Service Life May Require Replacement of an Upgraded System

We now consider the more interesting and perhaps more realistic case when equation 1 is not valid; that is, the service life of an upgraded system may require it to be replaced before the life cycle period expires. When this is true, the life cycle cost for upgrading the existing system in year 1, operating it for n years, and replacing it at the beginning of year $(n+1)$ is

$$LCC_u(L,n) = A_u + C_u \sum_{k=1}^{n} d^{k-0.5} + A_r d^n + C_r \sum_{k=n+1}^{L} d^{k-0.5}, \qquad 1 \le n < L \tag{8}$$

The four terms on the right side of the equation represent, respectively, the upgrade cost, the discounted cost of operating the upgraded system for n years, the discounted cost of purchasing the new system at the beginning of year $(n+1)$, and the discounted cost of operating the replacement system over the years $n+1$ through L.

For the special case in which a new system is bought in year 1 and there is no upgrade, we have

$$LCC_r(L) = A_r + C_r \sum_{k=1}^{L} d^{k-0.5}$$

$$= A_r + C_r \sum_{k=1}^{n} d^{k-0.5} + C_r \sum_{k=n+1}^{L} d^{k-0.5} \quad . \tag{9}$$

Note that we have deliberately written equation 9 so that its last term matches that of equation 8. This term represents the operating costs from year $(n+1)$ to the end of the life cycle period, L. For this period, operations will be with the replacement system under either alternative; therefore, we need not consider these years when comparing the life cycle costs.

Unless noted otherwise, from hereon we shall only consider the case of having to eventually replace an upgraded system before the life cycle period expires.

D. DETERMINATION OF MINIMUM LIFETIME FOR UPGRADING AN EXISTING SYSTEM

1. General Formalism

If there exists an $n^* < L$ such that $LCC_u(L, n) < LCC_r(L)$ for all $n \geq n^*$, then if the remaining lifetime of an existing system is at least n^*, it would be less costly to undertake an upgrade program and defer replacement until year (n^*+1) or after.

To determine the value of n^*, we equate equations 8 and 9 and drop the identical terms to get the equation

$$A_u + C_u \sum_{k=1}^{n} d^{k-0.5} + A_r d^n = A_r + C_r \sum_{k=1}^{n} d^{k-0.5} \tag{10}$$

Using the geometric series formula given in equation 6, we obtain the following relationship:

$$A_u + C_u d^{0.5} \frac{1-d^n}{1-d} + A_r d^n = A_r + C_r d^{0.5} \frac{1-d^n}{1-d}. \tag{11}$$

On collecting terms involving n on the left side and multiplying both sides by $(1-d)$, we have

$$(C_u - C_r)d^{0.5}(1-d^n) + (1-d)A_r d^n = (1-d)(A_r - A_u)$$

$$\left[(1-d)A_r - (C_u - C_r)d^{0.5}\right]d^n = (1-d)(A_r - A_u) - (C_u - C_r)d^{0.5} \qquad (12)$$

$$d^n = \frac{(1-d)(A_r - A_u) - (C_u - C_r)d^{0.5}}{(1-d)A_r - (C_u - C_r)d^{0.5}}$$

On taking the logarithm of each side of the above equation and solving for n, the number of years to defer replacement, we find the following crossover point, the point when the two alternatives have equal life cycle costs:

$$n^* = \frac{\ln\left[\dfrac{(1-d)(A_r - A_u) - d^{0.5}(C_u - C_r)}{(1-d)A_r - d^{0.5}(C_u - C_r)}\right]}{\ln d}. \qquad (13)$$

Equation 13 is one of the central results of this paper, a closed-form solution for the minimum lifetime needed by an upgraded system to be a cost-effective alternative to acquiring a new system now, when the residual values of the alternatives are omitted.

It should be noted that there are combinations of parameters for which no real, positive solution exists to equation 13. A solution is likely not to occur if the difference in O&S cost is large, the difference in acquisition costs is small, and the discount factor d is close to unity (discount rate i is close to zero). Such a case will usually result in a negative value for the logarithmic argument in the numerator of equation 13. That there is no solution for these parameters means that there never is a time for which upgrading is less costly than buying a new system under these conditions. Under these circumstances, buying a new system now would be preferred.

Note that the crossover point is independent of L, the life cycle period. For a given set of input values where there is a solution, if n is less than zero, or if n is greater than or equal to L, then the Replace-Now option is the preferred choice. If $1 \le n^* < L$, then Upgrade-First is the lower cost alternative *provided that the remaining lifetime of the upgraded system is at least* n^*. A solution of $n = 0$ occurs when $A_u = 0$; this special case is discussed below.

2. Example 1: No Residual Values

We consider a hypothetical system to gain insight into the general behavior of the life cycle equations and of equation 13, the solution equation. Consider the following inputs:

A_u = \$5 million	C_u = \$1.5 million	$i = 0.035$ (i.e. 3.5% discount rate)
A_r = \$35 million	C_r = \$1.0 million	$L = 20$ years

Figure 1 shows the graphs of the life cycle costs for the two cases: (1) upgrade first, then replace with new at year n, and (2) replace now (at year 1). It is seen that the Upgrade-First alternative shows a decrease in life cycle cost with increasing values of n and that the cost-equality point, in fact, does signify a crossover in favor of delaying the new acquisition. To solve for the crossover point, we apply equation 13 and find that $n^* = 8.14$, which agrees with that shown on the graph in Figure 1. Thus, we conclude for this example that if the remaining life of a current system after an upgrade is at least 8.14 years, then the Upgrade-First option is worthwhile pursuing. Again we note that we are assuming that the life cycle cost is the only decision criterion, that an upgraded system will not likely survive for 20 years, the life cycle period, and that differences in performance are not relevant to the decision.

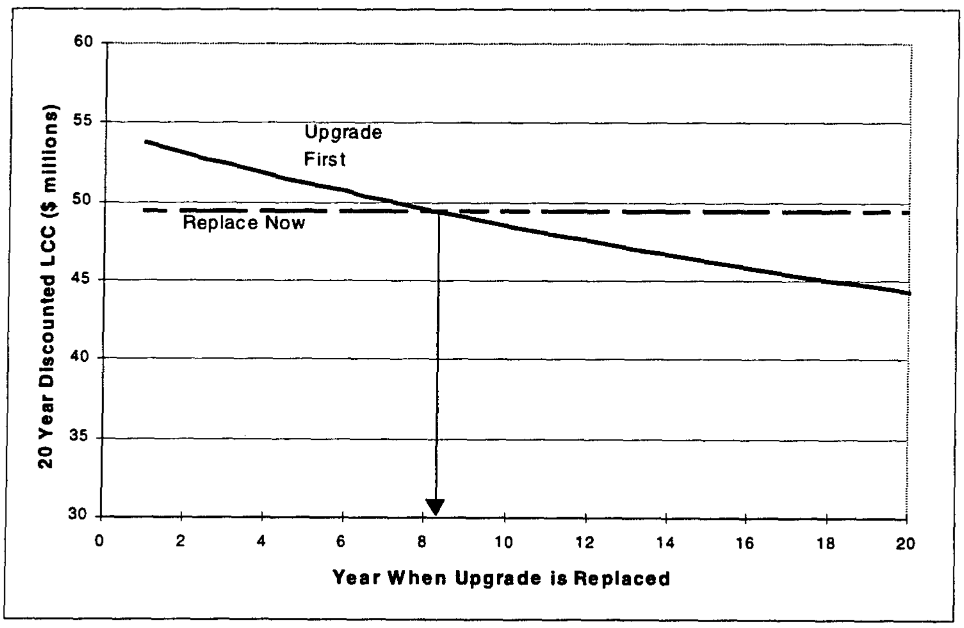


Figure 1. Comparison of Costs for Upgrade-First and Replace-Now Alternatives (Example 1, No Residuals)

Figure 2 shows the sensitivity of the results for a range of assumptions about discount rate and difference in O&S costs between the two options. Note that there is little variation within small excursions from the baseline value of 8.14 for the parameters selected in this example. For more extreme excursions, the needed lifetime increases more dramatically. As the difference in O&S costs increase and as the discount rate decreases, the value of n^* begins to rise dramatically. This is the expected behavior for the discount rate, since the value of deferring new acquisition depends on discounting to reduce the effective cost of the new system sometime in the future. The smaller the discount rate, the more years needed to break even.

The results are less obvious for O&S cost differences. One might at first think that the more costly the upgrade is to operate relative to the replacement, the fewer years one would want the upgrade flying. But this ignores the impact of discounting on the replacement acquisition cost. In fact, the more costly the upgrade is to operate relative to the replacement, the more years are needed to discount adequately the acquisition cost of the replacement in order to break even.

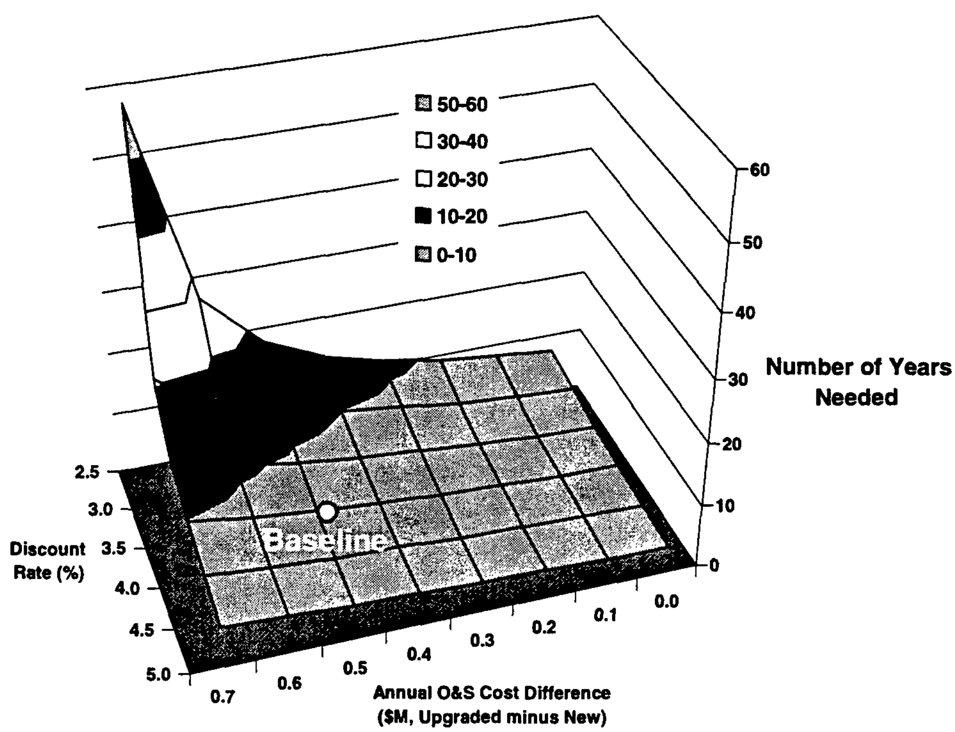


Figure 2. Dependence of Lifetime Needed on Discount Rate and Differences in O&S Costs

3. Selected Features of the Formalism

a. Monotonicity

We now consider the monotonicity of the life cycle cost equation for the Upgrade-First option. If it can be shown that *LCC(L,n)* is monotonically decreasing with *n*, then given a solution $n^* > 0$, we can be sure that operating the upgraded system for a number of years $n \geq n^*$ will retain the advantage of the Upgrade-First option. On taking the partial derivative of *LCC(L,n)* with respect to *n*, we have

$$\frac{\partial[LCC(L,n)]}{\partial n} = \left[A_r - \frac{(C_u - C_r)d^{0.5}}{1-d}\right] d^n \ln d \tag{14}$$

This result indicates that *LCC(L,n)* is monotonic with *n*, the direction depending on the term inside the brackets. Since $\ln d < 0$, (*d* is less than 1) the Upgrade-First life cycle cost decreases with *n* if

$$A_r > \frac{(C_u - C_r)d^{0.5}}{1-d} \tag{15}$$

Thus, if A_r is large relative to the operational cost difference adjusted by a discounting factor, then *LCC(L,n)* decreases with *n*. Any value of $n \geq n^*$ will yield savings over the Replace-Now option, and the savings increase with *n*.

By considering *boundary cases* for the variables, we can gain better insight into the mechanism underlying the Upgrade-First versus Replace-Now decision.

b. No Discounting

We first consider the case of not doing a present value analysis. This is equivalent to setting the discount rate, *i*, equal to zero, or setting the discount factor, *d*, to unity. For d=1 in equations 8 and 9, we get the following life cycle cost equations:

$$LCC_u(L,n) = A_u + C_u n + A_r + C_r(L-n)\,, \quad 1 \leq n < L \tag{16}$$

$$LCC_r(L) = A_r + C_r n + C_r(L-n) \tag{17}$$

We find that $LCC_u(L,n) < LCC_r(L)$ only if

$$\Delta C < \frac{-A_u}{n}, \; where \; \Delta C = (C_u - C_r) \tag{18}$$

Note that this inequality is independent of A_r. Because A_u is positive and ΔC is also (the operating cost of an upgraded system is greater than that of a new system), the inequality cannot be satisfied. Therefore, the Upgrade-First option cannot be less than the Replace-Now option when discounting is not applied.

c. Equal Operating Costs

It is possible in some cases that there is negligible difference in O&S costs for the upgraded and replacement systems. If ΔC is set equal to zero, we find from equation 13 the following solution for n^*:

$$n^* = \frac{\ln\left(1 - \frac{A_u}{A_r}\right)}{\ln d} \tag{19}$$

Since $A_u < A_r$, both logarithmic arguments are between 0 and 1, and the ratio of logarithms will always be positive; therefore, a solution for n^* will always exist and be greater than 0.

d. Equal Acquisition Costs

It should be obvious that if the upgrade and the replacement acquisition costs are the same, the Replace-Now alternative will always be preferred. If we rewrite the original life cycle cost equations for the case when A_u=A_r=A, we find that the Upgrade-First option has lower cost when

$$d^{n-1} < -\frac{\Delta C}{A}\sum_{k=1}^{n} d^{k-0.5} \tag{20}$$

Since d, ΔC and A are assumed to be positive, this inequality cannot be satisfied, thus reinforcing the intuitive notion that if an upgrade costs as much as a new system, one ought to buy the new system.

e. Continued Operation With the Existing System

As indicated earlier, this situation can be modeled by setting A_u=0. When this is done, we find from equation 13 that n^*=0, an ambiguous result implying either that a new system should be bought now or that one should operate the existing system for as long as possible. The correct decision depends on the monotonic direction of the life cycle cost, *LCC(L,n)*, which is determined through equation 15. If the direction is upward, replace now; otherwise continue operation with the existing system for as long as possible.

E. LIFE CYCLE COST MODEL WITH RESIDUAL VALUES

1. General Formalism

The preceding formalism failed to account for the residual value of the systems when they are retired and at the end of the life cycle period. We now address the inclusion of residual values. Three types of residual values are considered:

1. **Existing System**: If a replacement system is bought initially, the existing system may have some residual value, which we denote by $R_e(y)$, where y is the age of the system. For our purposes, we will assume that the age of the existing system does not significantly affect the residual value so that $R_e(y) = R_e$, a constant. This is not unreasonable since we are considering systems so old that they must either be upgraded or replaced. Therefore, the residual value will be small (perhaps equal to the system salvage value) and most likely will not be age-dependent. In this way, we avoid introducing another variable.
2. **Upgraded System**: If the existing system is upgraded and then replaced after n years of operation, it may have some residual value at that time. We denote this value by $R_u(n)$. This residual will have a discount factor reflecting n years applied to it.
3. **Replacement System**: The last kind of residual value involves the replacement system, which is assumed to be introduced at the beginning of year $(n+1)$. Since this system is new at that time, at the end of the life cycle period, L, it will have been in operation for L-n years, and we denote its residual value by $R_r(L-n)$. The applicable discount period is L years.

To incorporate residual values into the life cycle cost model for upgrading now and then replacing the upgrade in n years later, we subtract out the terms $R_u(n)d^n$ and $R_r(L-n)d^L$ from equation 8. To incorporate the residual values into the life cycle cost model for the Replace-Now option, we subtract out the terms R_e and $R_r(L)d^L$ from equation 9.

After modifying equations 8 and 9 to include residual values, we obtain the following life cycle cost equations:

$$
\begin{aligned}
LCC_u(L,n) &= A_u + C_u \sum_{k=1}^{n} d^{k-0.5} + [A_r - R_u(n)]d^n + C_r \sum_{k=n+1}^{L} d^{k-0.5} - R_r(L-n)d^L \\
&= A_u + d^{0.5} C_u \frac{1-d^n}{1-d} + [A_r - R_u(n)]d^n + C_r d^{0.5} \frac{d^n - d^L}{1-d} - R_r(L-n)d^L, \ n \geq 1
\end{aligned}
\qquad (21)
$$

$$LCC_r(L) = A_r - R_e + C_r \sum_{k=1}^{L} d^{k-0.5} - R_r(L)d^L$$

$$= A_r - R_e + C_r d^{0.5} \frac{1-d^L}{1-d} - R_r(L)d^L \qquad (22)$$

To find the crossover point, we set $LCC_u(L,n)$ equal to $LCC_r(L)$ after first defining the residual value functions R_e, $R_u(y)$, and $R_r(y)$. In general, the incorporation of residual value functions will prevent a closed-form solution since they typically depend on n. However, a solution is easily found by either plotting the life cycle cost functions and noting the crossover point or using an iterative numerical method such as provided by the Solver routine in Excel.

2. Example 2: Example 1 with Residual Values Included

This example is an extension of Example 1 but with residual values. We assume the following functional dependencies of the residuals. All costs are expressed in constant year millions of dollars:

$R_e = 0.1$	(\$100,000 is assumed to represent the salvage value of the system)
$R_u(y) = 0.6A_u\,(0.9)^y$	(assumes a 10 percent per year depreciation, starting with 60 percent of the upgrade cost)
$R_r(y) = 0.8A_r\,(0.9)^y$	(assumes a 10 percent per year depreciation, starting with 80 percent of the new system cost)

The graph of the two life cycle cost functions is shown in Figure 3. The crossover point is approximately at 3.7 years, less than half the value for the no-residual case. The inclusion of residuals reduces the lifetime required for the upgraded alternative to prove itself cost-effective.

At first this might seem counter-intuitive. One might argue that because the replacement systems have a much greater potential residual value than the existing or upgraded systems, introducing residuals into the analysis would more likely favor buying a replacement system over an upgrade and thus require an upgraded system to have a longer lifetime. However, by purchasing the new system now, one minimizes its relative residual value at the end of the life cycle period, i.e., a replacement now produces a 20-year-old system at the end of a 20-year life cycle. A deferred replacement results in a younger and more valuable system at the end of the same 20-year period. Therefore, delaying replacement system purchase by upgrading the existing system becomes even more beneficial than for the case when residuals are not considered. That is why, in order to show life cycle cost benefits, an upgraded system has to be able to survive only 3.7 years using residuals instead of 8.14 years without residuals.

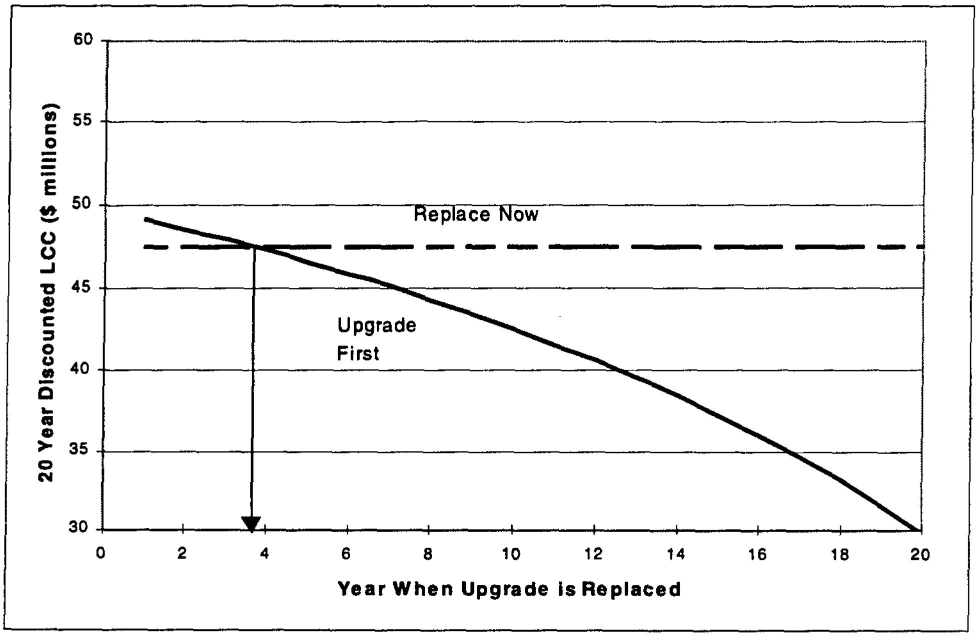


Figure 3. Comparison of Costs for Upgrade-First and Replace-Now Alternatives (Example 2, Including Residuals)

F. ADDITONAL CONSIDERATIONS AND EXTENSIONS OF THE FORMALISM

1. Time-Dependent Cost Functions

As discussed so far, the life cycle cost models are of a simple form, particularly with respect to using constant values for the replacement system cost, A_r, and the operating costs, C_u, and C_r. Since the replacement cost is incurred some time in the future for the Upgrade-First option and the operating costs occur over the life cycle period, the constancy assumption may not be valid. To provide greater generality to the model, we can designate these model inputs as time-dependent functions. This would not cause serious complications in the case of A_r, because that extension can be treated in the same way we treated the residual values. However, for the operating cost values, which are summed over the n years, the equations become a bit more complicated. Rewriting equation 8 to reflect time dependent cost functions, we have

$$LCC_u(L,n) = A_u + \sum_{k=1}^{n} C_u(k)d^{k-0.5} + A_r(n)d^n + \sum_{k=n+1}^{L} C_r(k-n)d^{k-0.5}, \quad 1 \le n < L \tag{23}$$

If we assume that operating costs increase by a constant percentage each year (sometimes termed "maintenance creep"), we can eliminate the summation terms. We shall let P_u and P_r represent the creep percentages for the upgrade system and replacement systems, respectively. Thus, for m years of operation under Case x, $x = u$ or r, we have

$$C_x(m) = C_x^o(1+P_x)^m, \tag{24}$$

where C^o represents the baseline cost, defined so that $C^o(1+P)$ is the operating cost in year 1.

This leads to the following life cycle cost equation for upgrading first and replacing in year $(n+1)$:

$$LCC_u(L,n) = A_u + d^{0.5}(1+P_u)C_u^o\frac{1-Q_u^n}{1-Q_u} + A_r(n)d^n + d^{0.5}(1+P_r)^{1-n}C_r^o\frac{Q_r^n - Q_r^L}{1-Q_r} \tag{25}$$

where

$$Q_x = (1+P_x)d, \quad x = u,\ r \tag{26}$$

Note that we define $A_r(n)$ to be the cost of the replacement system when it is purchased n years after the upgrade. The equivalent equation for the Replace-Now option is

$$\begin{aligned} LCC_r(L) &= A_r(0) + \sum_{k=1}^{L} C_r(k)d^{k-0.5} \\ &= A_r(0) + d^{0.5}C_r^o(1+P_r)\frac{1-Q_r^L}{1-Q_r} \end{aligned} \tag{27}$$

To determine the crossover point, if one exists, we can plot equations 25 and 27 or use numerical methods as we did for the case of residuals. If residuals are also to be considered, we must add the appropriate residual functions as we did to equations 21 and 22.

2. Example 3: Example 1 with Time Dependent Cost Functions

The cost values used in Example 1 were as follows:

$A_u =$ \$5 million $\quad C_u =$ \$1.5 million

$A_r =$ \$35 million $\quad C_r =$ \$1.0 million

For the replacement system acquisition cost, we will now assume that the cost will increase linearly by 0.5 percent for each year of delay so that

$$A_r(n) = 35[1 + 0.005\,n]$$

We will also assume for this example that the above operating costs are for the first year of operation and that the "creep" factor is 1 percent for the upgraded system and 0.5 percent for the replacement system. This leads to the following set of factors:

$$C_u^o = 1.5/1.01 = 1.463, \quad C_r^o = 1/1.005 = 0.995$$
$$Q_u = 1.01d = 0.976, \quad Q_r = 1.05d = 0.971$$

Using these inputs, the life cycle cost for the two alternatives can be plotted to determine the crossover point, if one exists, or a numerical method can be used. Figure 4 shows the results. Also shown on the graph, as dotted curves, are the previous results when the cost functions are not time dependent. For the latter, the crossover occurred at 8.14 years. With the time dependent factors we used, the life cycle costs increase, and the upgraded system now has to survive 11.03 years before that option shows a life cycle cost benefit. A result that may be counter-intuitive at first look is the following: if the new system acquisition cost increases more than 1.1 percent per year, (a one-half percent per year was used in the example), then one would not select the Upgrade-First alternative at all. The discounting benefit cannot balance the increased cost of the new system acquisition.

Note that, in general, with time-dependent cost, monotonicity of the life cycle cost function for the upgrade option is no longer assured. If we had set the maintenance creep of the upgraded system to be 2.5 percent instead of 1 percent, the associated life cycle cost decreases to about \$50.9 million at n=15 and then starts to increase. Therefore, the life cycle cost for the Upgrade-First option never gets below the Replace-Now life cycle cost of \$50.01 million. This is illustrated in Figure 5.

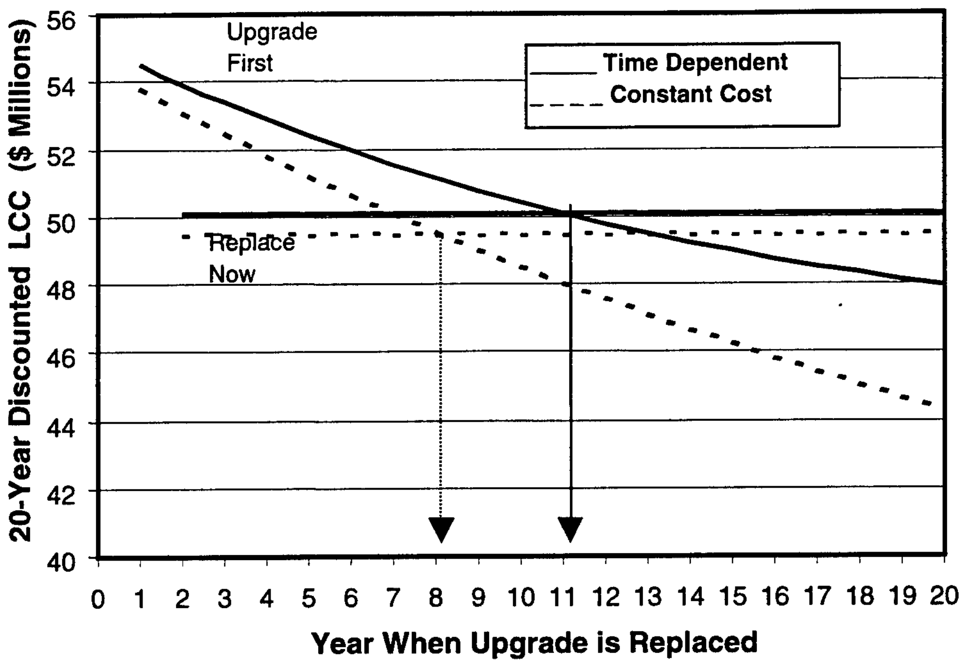


Figure 4. Comparison of Costs for Upgrade-First and Replace-Now Alternatives (Example 3, Time Dependent Cost Functions)

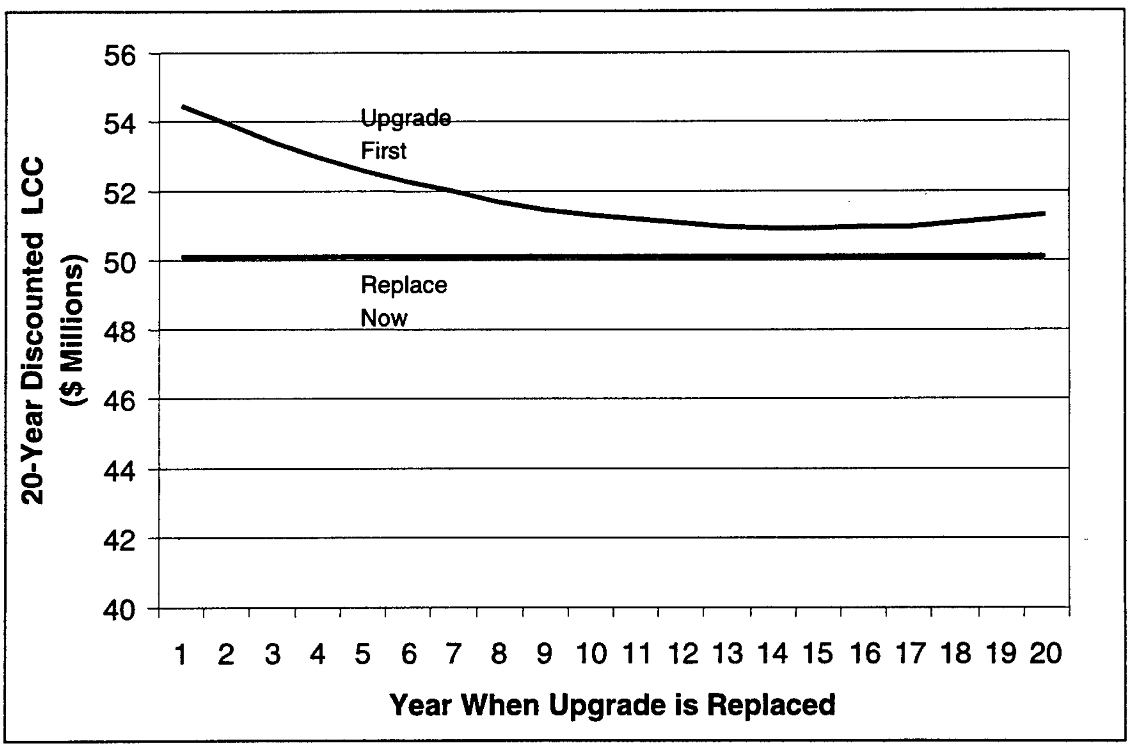


Figure 5. Illustration of Non-Monotonicity for Time-Dependent Cost Functions (Example 3, Time Dependent Cost Functions With 2.5% Upgrade Creep Rate)

3. Population Issues

a. Age and Modernization Factor Variability

Up to now we have considered only a single system. In this section, we consider issues related to a population of systems. We assume that there is a set of characteristics in the system population that, when considered in combination, require that the population be modernized through upgrade or replacement. This combined characteristic, herein termed the Modernization Factor, might well be represented by age, which often acts as a surrogate for one or more other characteristics. Other possibilities for the Modernization Factor are performance level, safety, commonality, and reliability.

The Single System Analysis, described earlier, would be valid if the population was homogeneous with respect to age as well as with respect to the system characteristic(s) defining the Modernization Factor. Two key questions are

- Can one set of inputs (acquisition/upgrade cost, O&S cost, residual values) be used for all systems in the population?
- Can one decision be made that applies to all the systems?

If the answers to both questions are affirmative, then the Single System Analysis approach is applicable. However, if there is variability in the population so that a single set of inputs is not applicable, then it is probably wise to subdivide the population into homogeneous groups and treat each group separately. In such a case, there may be some groups for which an upgrade versus replace decision can be deferred until a critical value is reached for the group's age or relevant system characteristic.

Assume now that the Modernization Factor is not age but a single set of cost inputs is appropriate. For example, a fleet of vehicles may have to be modernized to meet new emission and safety standards. Also assume that the ages or mileages of the vehicles show significant variation but that the upgrade, operating costs, and residual values do not vary significantly. In this case, a single system analysis may not be appropriate because the decision to upgrade or replace depends on the remaining life of the vehicle and such life is function of age. In this case, we can still find the minimum replacement time, n^*, to make the Upgrade-First option the better choice, and then system by system determine whether to upgrade or replace now. If the age of a vehicle is such that its remaining life is greater than n^*, then upgrade first and replace when end-of-life is reached. If the remaining life is less than n^*, then replace now. We formalize this below in establishing a replacement schedule for a population of systems with varying ages.

b. Replacement Schedule

The section deals with situations when the ages within a population or analysis group differ significantly but the population or group has been defined so that a single set of cost inputs is satisfactory. For this case the decisions involve a number of systems, each of differing age. This section derives formulae for the replacement schedule, given the age distribution of the systems under consideration, an estimate of the required replacement age, and a calculation of $n*$. The replacement schedule serves then as the programmatic schedule for planning future investments.

Consider N_0 systems. The current distribution of ages among these is given in general by the frequency function $N(\alpha)$, where α represents the age in years, and $N(\alpha)$ is the number of systems with age α. For convenience, we choose to restrict the ages to integer values, although the formalism can be readily extended to continuous functions. The frequency function satisfies the equation:

$$\sum_{\alpha=1}^{\infty} N(\alpha) = N_0 . \tag{28}$$

We now introduce a new, but related frequency function that depends not only on the age, but also on the number of years that have passed since upgrading. We define the function $N(n, \alpha)$ as the number of systems α years old, n years after all N_0 have been upgraded. All the upgrades are assumed to have been performed at the beginning of the same year, so

$$N(0,\alpha) = N(\alpha) . \tag{29}$$

The new frequency function $N(n,\alpha)$ represents a simple displacement of the initial frequency function $N(\alpha)$ along the age or α-axis by n years, if we assume that all systems within the distribution age at the same rate.[3] Thus, the frequency functions are related by the recurrence relations, which, in turn, result from the displacement relationships among the functions. Repeated iterations with the recurrence relationship,

$$N(n,\alpha) = N(n-1,\alpha-1) , \tag{30}$$

---

[3] The assumption that all members age at the same rate is reasonable if the "age" is chronological age, but we also mean it to represent "use age," such as flying hours for aircraft or miles driven by wheeled vehicles. If different members of the distribution are used at significantly different rates, the frequency function N(α) will change every year not by a simple translation but also by a deformation. The expressions derived in this section would then need to be generalized to allow for changes in shape.

yields a useful relationship between $N(n,\alpha)$ and $N(\alpha)$:

$$N(n,\alpha) = N(\alpha - n)\,. \tag{31}$$

The more general frequency function satisfies the equations

$$\sum_{\alpha=1}^{\infty} N(n,\alpha) = N_0\,, \tag{32}$$

and

$$\sum_{n=0}^{\infty} N(n,\alpha) = \sum_{r=1}^{\alpha} N(r)\,, \tag{33}$$

where the last expression makes use of equation 31 and the following:

$$N(\alpha) = 0\,,\ \text{if}\ \ \alpha \leq 0. \tag{34}$$

We introduce a new function, the replacement rate $R(n)$, which is the number of replacement systems to be acquired in year n, such that the acquisition of upgraded and replacement systems is conducted in the most cost-effective manner. The estimate of this variable is the point of this section of the paper. For this we need, in addition to the formal frequency functions just introduced, two additional parameters: the cost-effective crossover point (n^*) and the actual service life of the systems (SL).

Recall that the crossover year n^* represents the minimum number of years of remaining life an upgraded system must possess in order to be a cost-effective alternative to immediate replacement. In the formalism to follow, we use integer values for number of years, so the calculated n^* must be rounded up to the next larger integer if it is not an integer, as in the examples. Upgrading will replace some worn components, but there is still a limit to the life. We assume that all the systems have the same service life, SL, but, because of age variation, there is variation in remaining life after the upgrade. Some systems are, of course, closer to the service life limit than others, as the frequency function illustrates, but all are assumed to expire when their age reaches this limiting value. This lifetime limit would presumably be determined by test criteria such as the time at which there would be widespread fatigue cracking, massive component failures, or other life-limiting phenomena that would be too costly and timely to repair.

With these parameters, the number of replacements needed in year 1 is identical to the number of systems whose age equals or exceeds SL exactly n^* years after upgrading. In other words, $R(1)$ equals the number of systems that should not be upgraded when cost is considered, but should be replaced immediately. In mathematical terms using the frequency functions just introduced, the replacement rate in year 1 would be

$$R(1) = \sum_{\alpha=SL}^{\infty} N(n^*,\alpha) = N_0 - \sum_{\alpha=1}^{SL-1} N(n^*,\alpha). \tag{35}$$

Since there would be no additional replacements needed for n^* years (the oldest upgraded system that was not replaced has at least n^* years remaining in its life, by construction), the annual replacement rates must be zero throughout this period of time:

$$R(2) = R(3) = \ldots = R(n^*) = 0. \tag{36}$$

The replacement rates for the years beyond n^* are obtained by using the frequency functions and counting backward from SL. Specifically, they are:

$$\begin{aligned} R(n^*+1) &= N(n^*, SL-1), \\ R(n^*+2) &= N(n^*, SL-2), \\ &\ldots \\ R(SL-1) &= N(n^*, n^*+1), \text{ and} \\ R(n) &= 0 \; for \; n \geq SL. \end{aligned} \tag{37}$$

We can use equation 31 to simplify and express replacement rates in terms of the initial frequency function $N(\alpha)$ and the parameters n^* and SL.

$$R(1) = N_0 - \sum_{\alpha=1}^{SL-1-n^*} N(\alpha), \tag{38}$$

$$R(2) = R(3) = \ldots = R(n^*) = 0, \tag{39}$$

and

$$\begin{aligned} R(n^*+1) &= N(SL - n^* - 1), \\ R(n^*+2) &= N(SL - n^* - 2), \\ &\ldots \\ R(SL-1) &= N(1), \end{aligned}$$

and

$$R(n) = 0 \; for \; n \geq SL. \tag{40}$$

These constitute the basic replacement schedules in terms of the current age distributions, constrained by cost and the known lifetime of the systems being upgraded.

The replacement rates also must also satisfy the requirement that over the lifetime *SL*, all systems must have been replaced, either initially in year 1, or subsequently after an upgraded system was retired. This expression is:

$$\sum_{n=1}^{SL} R(n) = N_0 \ , \qquad (41)$$

and can easily be shown to be consistent with the set of equations derived above.

4. Example 4: Example 1 with Age Distribution

We illustrate the use of the replacement rate equations by an example. Using the standard example with $n^* = 8.14$ (rounded up to $n^* = 9$ for this section), assume the distribution of ages given in Figure 6 for 170 systems. For purposes of illustration, assume the limiting lifetime *SL* for each is 25 years. The distribution includes relatively new systems as well as a number older than $SL\text{-}n^*$. These older ones are a lighter color in the figure and should, by the criteria of this paper, be replaced in year 1, since their remaining lifetimes are less than n^*.

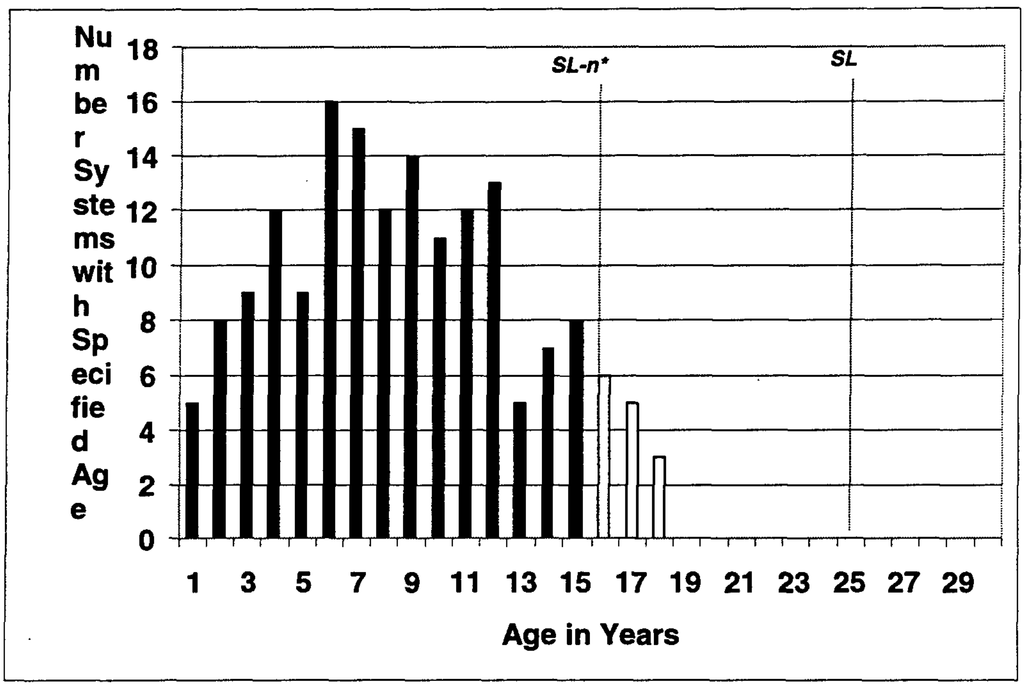


Figure 6. Age Distribution of Example Systems

The lowest cost replacement schedule consistent with this frequency is calculated from the set of equations and is illustrated in Figure 7. Note that the systems that exceeded the cutoff time of *SL-n** are all replaced in year 1, followed by a gap of n^* years before additional replacements are needed. The schedule for replacements thereafter becomes a mirror image of the original frequency function $N(\alpha)$.

While Figure 7 illustrates the most cost-effective replacement schedule for the example, other considerations outside those taken into account here may affect the actual schedule implemented. For planning purposes, a more level rate of replacements may be desired, both to keep annual expenditures for this purpose within designated bounds and to ensure an uninterrupted production rate for the industry producing the replacement systems. Thus the lengthy gap and the peaks and valleys shown in Figure 7 might disappear in an actual replacement schedule.

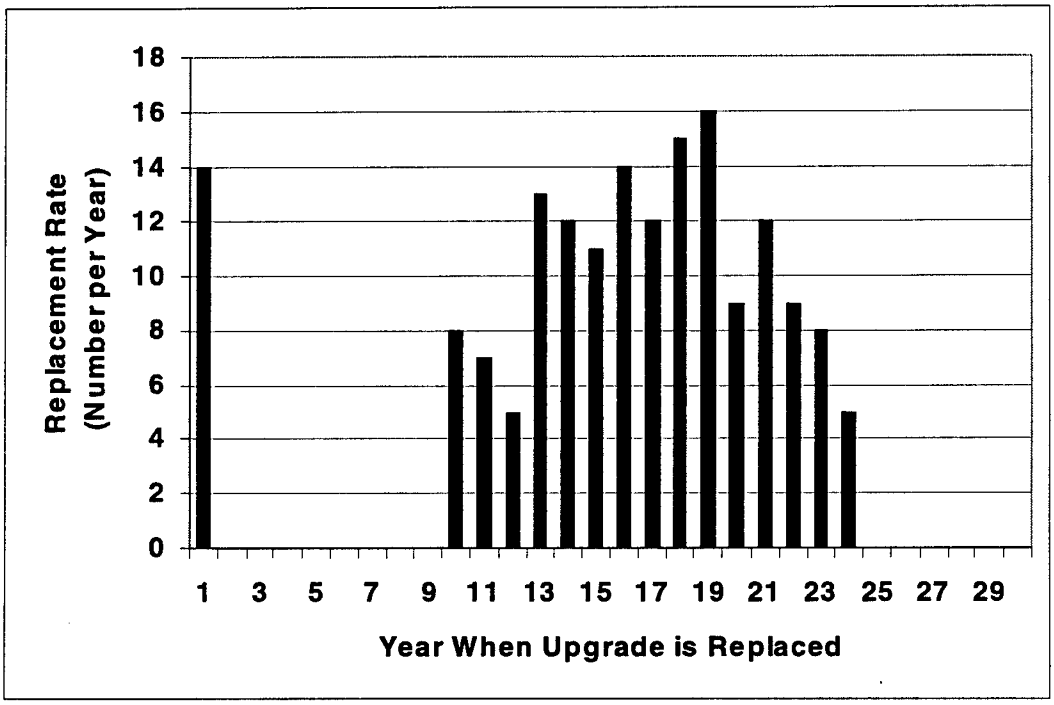


Figure 7. Optimum Replacement Schedule for Example Systems

5. Inclusion of Performance Differences

In this section we address the differences in performance expected from upgraded and replacement systems. We have assumed to this point that the differences in performance are irrelevant to the decision between the Upgrade-First and Replace-Now alternatives. In general we expect the replacement to be better than the upgrade in some measures, certainly in mission capable rate, and perhaps in other ways. We need to quantify the performance difference in order to proceed.

Let us consider N_u systems that are candidates for upgrading. When replaced, either now or ultimately, we shall assume that the difference in performance in the systems means that a different number, N_r, replacement systems are required to provide the same "performance" as the N_u upgraded systems. The specific definition of "performance" depends on the actual system being considered and is left unspecified in this general formalism. It could be the number of mission-capable systems. It could involve a specified level of firepower delivered or an upper limit on Blue casualties in a certain scenario. It could be the cargo-carrying capacity. It is open to interpretation as fits the problem. An example will be given later.

Our assumption about the nature of the performance measure permits us to establish a relationship between N_u and N_r as follows:

$$N_r = fN_u \tag{42}$$

where (usually) $0<f<1$. The relative performance factor f is a measure of the performance of the upgraded system relative to the replacement.

The life cycle cost for the equivalent of N_u systems is now obtained by a generalization of equation 1 with the insertion of N_u and N_r, where appropriate. The life cycle cost for upgrading N_r existing systems in year 1 and then replacing them all with N_r replacements in year n is

$$LCC_u(L,n) = N_u[A_u + C_u\sum_{k=1}^{n} d^{k-0.5}] + N_r[A_r d^n + C_r \sum_{k=n+1}^{L} d^{k-0.5}]. \tag{43}$$

For the special case where N_r replacement (new) systems are bought in year 1 and no existing systems are upgraded, we have the following revision to equation 9:

$$LCC_r(L) = N_r[A_r + C_r\sum_{k=1}^{L} d^{k-0.5}] \tag{44}$$

To determine n^*, we equate the two life cycle costs, use the relationship between N_r and N_u, indicated above, and solve as before to get a revised crossover point expression:

$$n^* = \frac{\ln\left[\dfrac{(1-d)(fA_r - A_u) - d^{0.5}(C_u - fC_r)}{(1-d)fA_r - d^{0.5}(C_u - fC_r)}\right]}{\ln d} \tag{45}$$

This equation is identical to equation 13 except that a new variable, the relative performance factor f, has been introduced as a multiplicative factor for the two replacement cost terms.

6. Example 5: Example 1 with Performance Differences

The sensitivity of results for n^* in Example 1 to variations in f is shown in Figure 8 below. The results are *extremely* sensitive to this factor. For a 10-percent decrease in performance for the upgrade relative to the replacement (i.e., f=0.9), the required lifetime jumps from 8.14 years to 12.78 years, a 57-percent increase. For a 20-percent decrease in performance (f=0.8), the required upgrade lifetime leaps to 30.80 years, a nearly 280-percent increase from the baseline value.

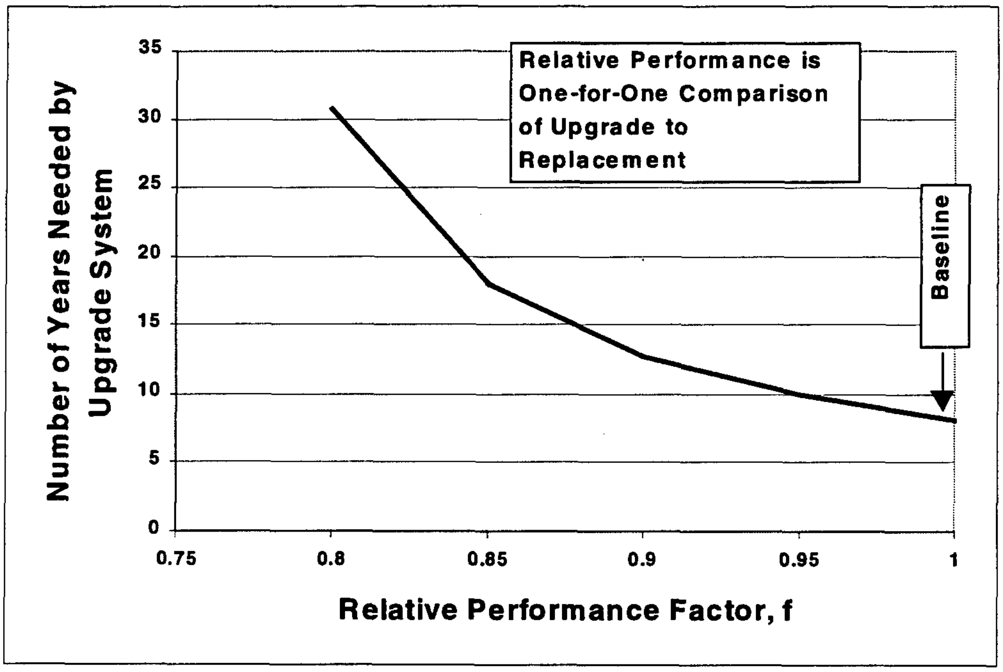


Figure 8. Dependence of Required Lifetime of Upgrade on Relative Performance Factor

If residual values are also included in the expanded formalism that treats differences in performance, equating the total life cycle costs for N_u upgraded systems and N_r replacement systems transforms equations 20 and 21 into the following solution equation for n:

$$N_u A_u + d^{0.5} N_u C_u \frac{1-d^n}{1-d} + [N_r A_r - N_u R_u(n)]d^n + N_r C_r d^{0.5} \frac{d^n - d^L}{1-d} - N_r R_r(L-n)d^L$$

$$= N_r A_r - N_u R_e + N_r C_r d^{0.5} \frac{1-d^L}{1-d} - N_r R_r(L)d^L. \quad (46)$$

This equation can be simplified by using the relative performance factor *f*. It is clear that the expressions obtained are identical to those derived without considering performance differences (i.e., equations 21 and 22) except that all cost terms associated with replacement systems—A_r, C_r, and R_r—are now multiplied by *f*. No general closed form solution can be given for n^*; either graphical or numerical means must be used.

7. Risk Issues

Life cycle cost analyses typically involve estimates of parameters that represent activities far in the future. Accordingly, there is usually considerable risk and uncertainty associated with LCC estimates. A common approach for dealing with risk is to assign distributions to the variables in the LCC equation and attempt to use statistical techniques to develop a distribution of the estimated costs. In many cases, the LCC model is too complex to perform this analytically, so approximations or simulations are used. The latter is especially easy to do with today's computer hardware and software capabilities.

To illustrate the simulation approach, we shall again use Example 1 in an Excel model employing Crystal Ball, a software simulation package that works with Excel spreadsheets. For ease in following the discussion, we repeat below equation 13, the solution equation for the crossover point, when all costs are constant, residuals are not used, performance differences are not an issue, and a single system analysis makes sense.

$$n^* = \frac{\ln\left[\dfrac{(1-d)(A_r - A_u) - d^{0.5}(C_u - C_r)}{(1-d)A_r - d^{0.5}(C_u - C_r)}\right]}{\ln d} .$$

We see that n* depends on the following five variables: d, A_u, A_r, C_u, C_r. Since the discount factor d is a direct function of the interest or discount rate, i, we shall refer to the latter in this section when discussing the variables. To use Crystal Ball, the variables in equation 13 that are to have probability distributions are specified in what is known as Assumption Cells. The equation for n^*, a function of these variables, is located in what is called a Forecast Cell. Before the simulation is started, distributions, where deemed appropriate, are assigned to each of the variables along with associated distribution parameters. If a variable is to have a normal distribution, for example, both a mean and a standard deviation are to be chosen by the analyst. Truncation points can be defined for a distribution as well. Discrete distributions can also be used. Once the distributions are defined, the user indicates how many simulation trials are to be conducted. With a simple problem like this, 10,000 trials can be run in just several minutes using a typical Pentium personal computer. The software keeps track of the results, generating such statistics as mean, median, mode, variance, range, and frequency distribution of the values in the Forecast Cell(s).

One concern in running a simulation is that a combination of the randomly selected values for the variables could lead to an improper mathematical operation such as dividing by zero, or, in this example, the more likely case of trying to take the logarithm of a negative number. Therefore, the spreadsheet should be designed to check intermediate results to avoid this so that the simulation is not halted each time an improper operation is attempted.

8. Example 6: Example 1 with Risk Assessment

Figure 9 shows the distributions we used for Example 1 with illustrative brief explanations for the choices. Also shown in the figure are the observed means after a 10,000 trial run was performed. This particular figure was extracted from the summary report developed by the Crystal Ball software and modified to include the explanations. After using these distributions in the Excel/Crystal Ball model, the key results shown in Table 2 were obtained.

Table 2. Simulation Results for Sample Problem

Factor	Result
Probability that a crossover will be found between 1 and 20 years.	0.728
Probability the Replace-Now option is preferred	0.272
Average crossover point, given a crossover	8.05 years

We see from the table that the estimated probability that the Upgrade-Now option is preferred is approximately 0.73. The complement, 0.27, is a reasonable measure of the risk associated with upgrading. In other words, if the Upgrade-First option was selected through application of equation 13 using average values, 27 percent of the time the variables in the problem will turn out to have values such that the Replace-Now option would have been a better choice. The average crossover point of 8.05 years obtained through the simulation, given that a crossover occurs, compares favorably with the value of 8.14 years, obtained when constant values were assumed for the inputs.

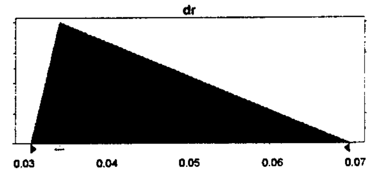
Assumption: Discount Rate, i

Triangular distribution with parameters:

Minimum	0.03
Likeliest	0.04
Maximum	0.07

Mean value in simulation was 0.05

Do not expect rate to go below 3% but much higher values possible- so right tailed distribution is reasonable.



Assumption: Upgrade Cost, Au

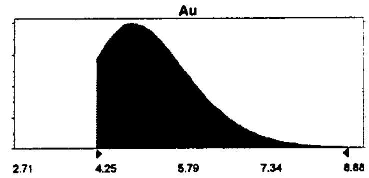
Lognormal distribution with parameters:

Mean	5.00
Standard Dev.	1.00

Selected range is from 4.00 to +Infinity

Mean value in simulation was 5.25

Right-tailed distribution because of possible structural changes. Do not expect costs to go below $4 million.



Assumption: Replacement Cost, Ar

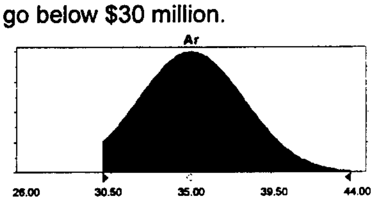
Normal distribution with parameters:

Mean	35.00
Standard Dev.	3.00

Selected range is from 30.00 to +Infinity

Mean value in simulation was 35.34

Symmetrical distribution appears reasonable. Do not expect cost to go below $30 million.



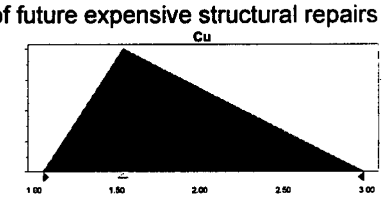
Assumption: Annual O&S Cost, Upgrade, Cu

Triangular distribution with parameters:

Minimum	1.00
Likeliest	1.50
Maximum	3.00

Mean value in simulation was 1.83

Right tail because of possibility of future expensive structural repairs



Assumption: Annual O&S Cost, Replacement, Cr

Normal distribution with parameters:

Mean	1.00
Standard Dev.	0.10

Selected range is from 0.85 to +Infinity

Mean value in simulation was 1.01

Symmetrical underlying distribution seems reasonable.

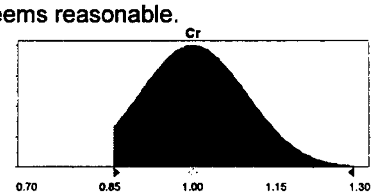


Figure 9. Distributions of Inputs Used in the Simulation

G. CONCLUDING OBSERVATIONS

We have devised a general formalism within which to address the question of whether to defer replacements through upgrades or to replace immediately. The formalism is of sufficient generality that it applies to a large set of acquisition problems confronting the Department of Defense and other system users. The initial set of conditions involving only acquisition and operating costs was extended to include residual values, performance differences, time-dependent costs, and population issues. Sensitivity of the model to various inputs were displayed and, in some cases, significant differences in results occurred for rather small changes, e.g., the discount rate and differences in performance. As with any life cycle cost analysis, there usually is a concern about the accuracy of costs and cost-related factors used in the model, especially expenditures far in the future. The last section, Risk Issues, offers one means to address this concern.

Appendix A

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This paper, supported by the Central Research Program at the Institute for Defense Analyses (IDA), establishes important decision rules for comparing upgrading existing systems to buying new ones. It addresses the important issues crucial to deciding whether to upgrade or buy a new system, with an emphasis on cost and the expected or known structural lifetime of the upgraded system. If the lifetime is unknown, the methodology can also be used to estimate the lifetime needed to warrant an upgrade vice a replacement with a new system. Finally, the methodology allows an acquisition/upgrade program to be defined for the future that would keep costs to a minimum. The methodology is sufficiently general to apply to a wide range of systems.

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